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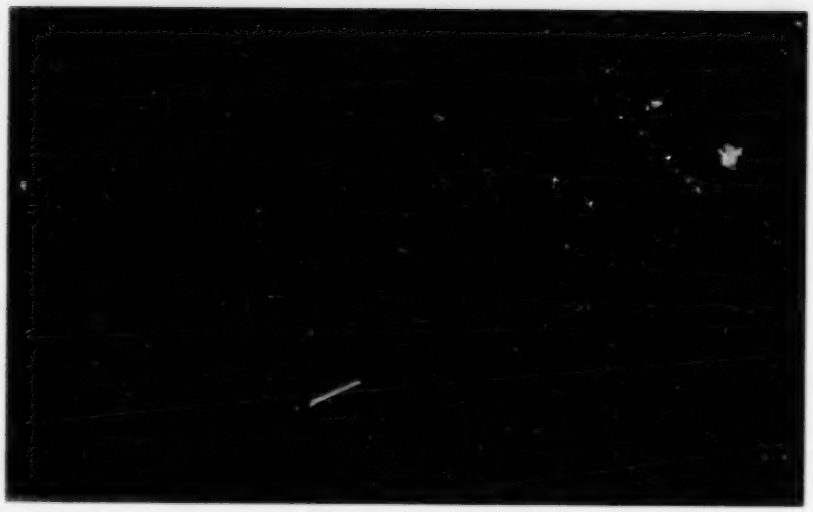

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Real-Time Scheduling on a Transit Bus Route: A 0–1 Stochastic Programming Model

by

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ABSTRACT

We consider a scheduling problem in real-time on a single transit bus route where buses may be express and stop only at a few places or turn before the terminus in order to respect the schedule. The problem is formulated as a 0–1 stochastic programming model with simple recourse in which the objective function accounts both for deviations from the schedule and for unsatisfied passenger demands. An application to sample data from the Shanghaï Transit company is described.

Keywords: Real-Time, Transit Scheduling, Stochastic Programming.

RÉSUMÉ

Nous étudions le problème de dresser en temps réel l'horaire d'une ligne d'autobus unique, quand certains des autobus sont des express qui ne s'arrêtent qu'à certains arrêts, ou qui peuvent revenir en sens inverse avant d'atteindre le terminus. Ce problème est formulé comme un modèle de programmation stochastique binaire avec recours simple dans lequel la fonction économique tient compte à la fois des déviations de l'horaire et des demandes de transport non satisfaites. Nous décrivons une application du modèle à des données provenant de la compagnie de transport en commun de Shanghaï.

Mots-clés: Temps réel, horaires de transport en commun, programmation stochastique.

1. INTRODUCTION

We consider the scheduling problem for the operation of transit bus and trolley lines in Shanghai, China. The problem is different from the one encountered in Western countries because the number of buses is not sufficient to satisfy the transit demand. Moreover because of congestion on the street, bus running times vary significantly which would necessitates large layover time to respect any planned schedule.

However, because the supply of buses is insufficient, it is not reasonable to plan for large layover at the terminus of bus lines, since this would be wasting transit capacity. In addition, to increase further the capacity of bus lines, all buses on a line do not always stop at every bus stop. Some buses are express and stop at only a few places. Some buses will not restart from the terminus, but at other points on the line to provide capacity to passengers waiting at intermediate points. Some buses will make short turns, meaning that they do not continue to the terminus, but turn back before the end of the line. These special trips have to be made at certain times of the day to respond to the travel demand.

Because of the congestion and lack of bus capacity, it is not really possible to efficiently plan the bus schedule in advance. There is thus a need for a system to

control the schedule in real time. The scheduling problem in this context can be decomposed into two subproblems: the operational planning of the schedule and real time control of it.

In operational planning, the problems are the following:

a) we must decide how many vehicles to allocate to each bus line in order to equilibrate service among bus lines given that there is a shortage of buses;

b) for each bus line, we must decide which patterns of services (series of stops) should be performed according to the time of day and predicted travel demands;

c) we must prepare an approximate bus driver schedule.

In real time control, the problem is to decide which pattern of services a bus leaving the terminus must perform given the operational plan, the previous decisions taken and the anticipated decisions for the future.

In order to achieve that objective, it will be necessary to estimate (if no real time data is available) the location of buses or trolleys on the line. To test the various strategies and validity of the model, a simulation of a line operation will have to be developed. This will be the subject of a future paper. In another paper, methods to generate a pre-determined bus schedule have been developed[11]. In this paper we present our approach for the real-time control problems.

The remainder of this paper is organized as follows. The problem definitions and the literature review are presented in section 2. Section 3 formalizes the problem as

a model of zero-one Stochastic Linear Integer Programming with Simple Recourse (SLIPSR); solution methods for solving SLIPSR problems are also briefly described. An application of the approach is discussed in section 4. Section 5 concludes the paper.

2. PREVIOUS WORK

Routing and scheduling of transit vehicles and crews has been a subject of interest for some time now and literature on the problem has been growing rapidly over the last decade. General surveys of routing and scheduling of vehicle and crews were made by Bodin et al.[2] and by Rousseau[17]. In his survey, Rousseau classified problems according to the various levels of planning. Strategic planning is related to long term commitments such as subway and light rail development and also modifications to the bus route structure. Operational planning problems are concerned with all the scheduling-related activity (see Scott[19] for an extensive study on the these problems). Ceder[3] presented an optimal design of transit short-turn trips to minimize fleet size where trips are initiated beyond the route departure point, or terminated

before the route arrival point, or both. Several bus scheduling software packages are available, some of these integrated into a crew scheduling software (see Wren[20], Rousseau[16], Daduna and Wren[5], and Desrochers[7]). Most of this research focuses on planning or pre-scheduled operational problems. The real-time scheduling problem in public transit, however, has received little attention. Cury et al.[4] presented a system for real time control of the underground railway system of Sáo Paulo. They established an analytical model to describe the flow of passengers boarding and leaving each train, the motion of trains, and the quality of performance. The model also minimizes the total deviations from a preestablished schedule. A decentralized control methodology was used by them for the "on line" control of train traffic in underground railway systems. In addition to this work, a number of transit queuing and simulation models have been developed by Bly and Jackson[1], Koffman[10] and Ryan[18] to evaluate the impact of different dispatching alternatives.

In this paper we are concerned with a different problem. Due to heavy traffic congestion and a shortage of bus capacity, delays of bus arrivals at the terminus occur frequently with the delay time varying from 5 to 30 minutes where the service headways are relative short (2-4 minutes on the average). Occasionally, therefore, the bus running times must be adjusted by skipping a number of stops in order to reestablish the desired headways. To achieve this, one has to take corrective strategies to minimize total penalty cost of unsatisfied passenger demands and make up the time

on a preestablished schedule in order to provide the best service to public. We now describe a formal mathematical model to address this problem.

3. PROBLEM FORMULATION AND MODELLING

3.1 Introduction

In real time scheduling, we must determine the bus skipping patterns. Suppose there are N_s stops along a line, and N_b buses departing in a given time period. Then, there are $(N_s \times N_s)/2$ skipping patterns for each bus (x_{ijk} , $i=1,...,N_b$; $j=1,...,N_s-1$, $k=j+1,...,N_s$) where x_{ijk} is a 0-1 variable indicating if the i^{th} departing bus travels directly from stop j to stop k. The optimization of bus dispatching is to determine the values x_{ijk} in order to provide the best service to the public.

The deviations of bus arrival times from the schedule and unsatisfied passenger demands are very difficult to represent with linear functions. Another difficulty is the number of 0-1 variables. Because of these difficulties, we have to select one of the following strategies:

1) Simplify the model so that it can be solved easily (by linear programming for

example).

2) Use an accurate model but solve the problem using approximate methods.

Alternative 1 has the advantage that we can solve the problem optimally. However, since the model is not accurate enough, this alternative cannot represent the real problem correctly and the solution obtained would not be useful to solve the problem in real time.

Alternative 2 is more practical. In real life dispatching, the main consideration is how to correctly evaluate the current situations and give quick responses. We now develop such an approach.

3.2 Problem Approach

We assume that the number of buses which can interact in the dispatching process is limited by a constant N_b. We assume also that the N_b^{th} bus will be on schedule and will not skip any stop. This is a realistic assumption, because we only need to control a limited number of dispatching buses to eventually get back on schedule. We thus propose the following rolling-horizon scheme:

- Solve the dispatching problem for the next n buses, which means finding the pattern of stops performed by these buses (except for the last one which will be the "full" pattern, since this bus is assumed to be on schedule).
- Use results from the model to dispatch only the next bus arriving at terminus

- Re-solve the problem for every bus arriving at the terminus.

3.3 Assumptions

The problem of real-time scheduling could be very complex. To simplify problem solving and presentation, we make the following assumptions.

1) Passengers at a stop may take several bus routes to their destination. We assume that the proportion of passenger taking each bus route can be calculated from a ridership survey and that it is not dependent upon the real-time scheduling decisions.
2) No allowance is made for any bus to pass other buses. This is certainly true for trolley lines. This assumption should eventually be relaxed.
3) Passengers will wait at their stop until a bus arrives (nobody leaves the system without taking the bus).
4) We assume that the last bus in the dispatching problem leaves on time and does not skip any stops.
5) We are working on a circular line that starts at stop 1 and finishes at stop 1. That is, there is only one control point (terminus) on the line at stop 1. This assumption does not limit the generality of the model.

3.4 Mathematical Model

The problem can be formulated as follows, Let

$$x_{ijk} = \begin{cases} 1 & \text{if bus } i \text{ travels from stop } j \text{ to stop } k, \\ 0 & \text{otherwise;} \end{cases}$$

d^+_{ij} = number of minutes bus i arrives earlier than the schedule at stop j;

d^-_{ij} = number of minutes bus i arrives later than the schedule at stop j;

y^+_{ij} = unsatisfied demand by bus i at stop j;

y^-_{ij} = unused capacity of bus i at stop j, where $y^-_{i0} = U_i$;

and

a_{ij} = arrival time of bus i at stop j;

b_{ijk} = number of passengers boarding on bus i and travelling from stop j to stop k, where $b_{i0k} = 0$;

c^+_{ij} = cost per minute of earlier arrival time of bus i at stop j;

c^-_{ij} = cost per minute of later arrival time of bus i at stop j;

π_{ijk} = random demand waiting for bus i and travelling from stop j to stop k;

N_b = number of buses to be rescheduled;

N_s = number of stops on the transit route;

p^+_{ij} = penalty per person of unused capacity of bus i at stop j;

p^-_{ij} = penalty per person not taken by bus i at stop j;

t_a = average alighting time per passenger;

t_b = average boarding time per passenger;

t_l = time lost during acceleration and deceleration;

t'_{jk} = bus travel time from stop j to stop k,where $t'_{0k} = x_{i0k} = 0$;

T_{ij} = scheduled arrival time of bus i at stop j;

U_i = the maximum passenger capacity of bus i.

The objective function is to minimize the total cost of deviations from the times on the schedule and the penalty cost of unsatisfied passenger demands.

$$\textit{minimize}\ z = \sum_{ij} (c^+_{ij} d^+_{ij} + c^-_{ij} d^-_{ij}) + \sum_{ij} E(p^+_{ij} y^+_{ij} + p^-_{ij} y^-_{ij}) \tag{1}$$

subject to:

1) Deviations of bus arrival time from the schedule:

$$a_{ij} + d^+_{ij} - d^-_{ij} = T_{ij} \quad i=1,\ldots,N_b,\ j=1,\ldots,N_s \tag{2a}$$

$$a_{ij} = a_{i,j-1} + t_a \sum_{q<j-1} b_{i,q,j-1} + t_b \sum_{k>j-1} b_{i,j-1,k} + t'_{j-1,j} + t_l\, x_{i,j-1,j} \tag{2b}$$

Since the bus arrival time a_{ij} in (2a) can be substituted by (2b), we have:

$$a_{i,j-1} + t_a \sum_{q<j-1} b_{i,q,j-1} + t_b \sum_{k>j-1} b_{i,j-1,k} + t'_{j-1,j} + t_l\, x_{i,j-1,j} + d^+_{ij} - d^-_{ij} = T_{ij} \quad i=1,\ldots,N_b,\ j=2,\ldots,N_s \tag{2}$$

2) Each bus exactly takes one skipping patten at stop j:

$$\sum_{k>j} x_{ijk} = 1 \quad i=1,\ldots,N_b,\ j=1,\ldots,N_s-1 \tag{3}$$

3) Number of boarding passengers is limited by the bus skipping pattern:

$$b_{ijk} \le U_i\, x_{ijk} \quad i=1,\ldots,N_b,\ j=1,\ldots,N_s-1,\ k=j+1,\ldots,N_s \tag{4}$$

4) Sum of boarding passengers is limited by the available capacity:

$$\sum_{k>j} b_{ik} \le y^{-}_{i,j-1} + \sum_{q<j} b_{iq} \quad i=1,\ldots,N_b,\ j=1,\ldots,N_s-1 \tag{5}$$

5) Deviations between allowed boarding passengers and the random demand for bus i at stop j plus passengers left by but i-1:

$$\sum_{k>j} b_{ik} + y^{+}_{ij} - y^{-}_{ij} - \sum_{k>j} \pi_{ik} + y^{+}_{i-1,j} \quad i=1,\ldots,N_b,\ j=1,\ldots,N_s-1 \tag{6}$$

6) Non-negative limitations

$$all\ \ x_{ik} = 0, 1;\ \ d^{+}_{ij}, d^{-}_{ij} \ge 0;\ \ y^{+}_{jk}, y^{-}_{jk} \ge 0 \tag{7}$$

Obviously, the model above is a 0-1 stochastic linear integer programming problem with simple recourse.

3.4 Solution Methods

Two-stage stochastic linear programming with simple recourse was first introduced by Dantzig in 1961 for linear programs with random coefficients for decision making under uncertainty. Let X be the set of those $\mathbf{x}$ vectors which have the property that for every π in the set of its possible values, there exists at least one $\mathbf{y}$ (depending on $\mathbf{x}$ and π) such that $T\mathbf{x} + I\mathbf{y} = \pi$. The model of two-stage linear programming under uncertainty is formulated by

$$\begin{aligned} &\textit{minimize} && \mathbf{c}'\mathbf{x} + E_{\pi}[\Phi(\mathbf{x},\pi)] \\ &\textit{subject to} && A\mathbf{x} \ge \mathbf{b} \\ & && T\mathbf{x} + I\mathbf{y} = \pi \\ & && \mathbf{x} \ge 0,\ \ \mathbf{y} \ge 0 \end{aligned} \tag{8}$$

where **c** and **x** are n-component vectors, **y** and π are $m2$-component vectors, T is a n x $m2$ matrix, I is an identity matrix, **b** is an $m1$-vectors, and A is an n x $m1$ matrix. $\Phi(\mathbf{x},\pi)$ is the optimum value of the following linear programming problem (in which **x** and π are supposed to be fixed for a moment)

$$\begin{aligned} &\textit{minimize} && \mathbf{p'y} \\ &\textit{subject to} && T\mathbf{x} + I\mathbf{y} = \pi \end{aligned} \tag{9}$$

This is called the second stage problem and (8) the first stage problem. The essential procedure of this algorithm is: the decision **x** is made, the values of the random variables occur, and then decisions **y** are made to compensate for any stage-one infeasibility. We solve the first stage problem only once but then the "experiment" producing the random variable π is "performed" many times. Thus we solve the second stage problem many times and $E_{\pi}[\Phi(\mathbf{x},\pi)]$ is interpreted as the arithmetic mean of the obtained optima. If π has a discrete distribution, then we can obtain at once the first stage and the second stage optimal solutions.

When decision variable x in (8) is limited by 0 or 1, we have 0-1 SLIPSR problems which can be formulated by

$$\textit{Minimize: } z=\sum_{j=1}^{n} c_j x_j+\sum_{i=1}^{m} E[p_i^+ y_i^+ + p_i^- y_i^-] \tag{10}$$

$$\textit{subject to: } \sum_{j=1}^{n} a_{ij} x_j \geq b_i\,, \quad i=1,\ldots,m1 \tag{11}$$

$$\sum_{j=1}^{n} t_{ij} x_j + y_i^+ - y_i^- = \pi_i\,, \quad i=1,\ldots,m2 \tag{12}$$

$$\text{all } x_j=0 \text{ or } 1,\ y_i^+, y_i^- \geq 0 \qquad (13)$$

where

c_j -- coefficient of x_j in objective function

a_{ij} -- coefficient of x_j in the i^{th} constraint

b_i -- the i^{th} deterministic resource

π_i -- the i^{th} stochastic resource with a joint probability distribution $F(b)$

$\mathbf{x}$ -- original decision vector

$\mathbf{y}^+$ -- shortage vector

$\mathbf{y}^-$ -- surplus vector

$\mathbf{p}^+$ -- shortage penalty vector

$\mathbf{p}^-$ -- surplus penalty vector

Thus far we have developed some mathematical properties with respect to the solution set of SLIPSR[13] [14]. Using these results we initially intended to develop an algorithm to obtain an optimal solution to the SLIPSR problem using a branch-and-bound approach[6] [12].

However, even though the feasible set for a SLIPSR is finite, the size of the problem that can be solved will be limited when using an exact algorithm, since the complexity of the procedure will be still $O(2^n)$. Heuristic approaches may therefore be more practical. We have developed two such methods for the SLIPSR problem:

HUSI and TABU. These methods which are described in detail in two other papers[13] [14] are capable of handling problems with hundreds or thousands of 0-1 variables.

4. AN APPLICATION

To evaluate the model described in the previous section, it was applied to data provided by the Shanghai transit company. Before describing the specifics of the Shanghai situation, we recall some classical results relative to bus travel times and passenger loads which are key inputs to our model.

4.1 Bus travel time

t'_{jk} depends on the road conditions and it usually belongs to a truncated Gamma distribution $\Gamma(\alpha,\beta,\tau)$ (Ryan[22]), where the parameter β does not vary from stop to stop while α and τ can be determined by observation for each pair of stops.

4.2 The number of passengers travelling from stop *j* to stop *k*

Özekicii[18] reported that passengers arrive at a stop according to a non-stationary

Poisson distribution. In other words, the passenger behave according to a pre-specified schedule if the headway interval length is large enough. In our case, because of a high frequency of bus service (2-4 minutes on the schedule or with random delay), the number of passengers arriving at a stop can be simply treated as a Poisson distributed random variable. That is, the expected number of passengers arriving at stop j travelling to stop k during a time interval t equals to the arrival rate of such passengers multiplied by t.

4.3 Data Collected

4.3.1 Passenger flows

Since the passenger flow volume is tremendously high in Shanghai (on a single trolley bus route 9, there are more than 50,000 passenger-trips per day), we divide the day into several time period within which we assume the passenger arrival rate to be constant; then for each time period t, we make a sample survey of the flow from stop j to stop k denoted as $P(t, j, k)$ by asking each passenger at his alighting stop, where he boarded. With this passenger flow information, we can derive:

$$\textit{the number of passengers alighting at the } k^{th} \textit{ stop during time period} = \sum_{j<k} P(t\ j\ k)$$

We can also calculate approximately, by adjusting for travel time,

$$\textit{the number of passenger arriving at stop } j \textit{ during the } t^{th} \textit{ time period} = \sum_{k>j} P(t\ j\ k)$$

4.3.2 Bus travel time

We have access to on-board surveys of bus travel time in which a checker writes down the bus arrival time and departure times at each stop, as well as the number of passengers getting on and off the bus. The bus travel time can be obtained by subtracting the bus arrival time from its departure time at the previous stop and the fixed time due to the acceleration and deceleration.

Due to the influence of weather, seasons and holidays, those passenger flows and bus travel times may vary from day to day, so that we need a number of surveys under different circumstances to obtain good estimates for these.

4.4 Solution Procedure

The solution procedure to solve the problem is:

- The two general heuristics for SLIPSR
- Empirical method

In addition to the two general heuristics for SLIPSR problems mentioned before, the problem was solved suing a specific heuristic based on empirical dispatching data.

This method stores a number of bus running patterns which have been most frequently used in practice. Those patterns may incorporate other restrictions coming from company policy. A solution will be selected from those prepared patterns and evaluated by the model with the estimated data.

Procedure:

Get an initial feasible solution $\underline{s} = (Q)$ by selecting one pattern for each bus, where $Q=\{q_i \mid q_i$ is the pattern selected for bus $i\}$, $P=\{p_k \mid p_k$ is a potential pattern$\}$

Let $BEST = W(\underline{s})$.

1) **Calculate** the feasible reduction $R_{ik} = BEST\text{-}W(s)$ for each pair of $(q_i, p_k), q_i \in Q$, $p_k \in P$, , where s is a new solution obtained when q_i is replaced by p_k. Set $R_{max} = \max\{R_{ik}\}$, if $R_{max} < 0$, terminate. Otherwise:

2) **Replace** q_i^* by p_k^* in Q, where p_k^* and q_i^* are the patterns resulting in R_{max}, and set $\underline{s} = s^*$, $BEST = W(\underline{s})$, then return to 1); or terminate if a satisfactory solution has been reached.

4.5 Numerical Results

The following numerical results were obtained by using data from actual bus operations. At time 17:06:50, three buses arrive at almost the same time at the terminus. The average headway is 2 minutes at that time. The different solutions are shown in Table 1, the total cost if no stops were to be skipped by these three buses is 5837 units.

TABLE 1. Solutions of the problem solved in different ways.

					Skipping Pattern									
No.	Volume	Arrival	Depart	Stop:	1	2	3	4	5	6	7	8	9	10
1.	180	6.50	7.00		1	0	0	0	0	0	1	1	1	1
2.	180	7.40	8.00		1	1	0	1	0	1	1	1	1	1
3.	180	8.55	9.00		1	1	1	1	1	1	1	1	1	1

Total cost: 5133 units.

a. Manual solution.

					Skipping Pattern									
No.	Volume	Arrival	Depart	Stop:	1	2	3	4	5	6	7	8	9	10
1.	180	6.50	7.00		1	0	0	0	0	0	1	1	1	1
2.	180	7.40	8.00		0	1	1	0	0	1	1	1	1	1
3.	180	8.55	9.00		1	1	1	1	1	1	1	1	1	1

Total cost: 4753 units.

b. Method 1 (HUSI) solution.

					Skipping Pattern									
No.	Volume	Arrival	Depart	Stop:	1	2	3	4	5	6	7	8	9	10
1.	180	6.50	7.00		0	0	0	0	0	0	1	1	1	1
2.	180	7.40	8.00		1	1	0	0	1	1	1	1	1	1
3.	180	8.55	9.00		1	1	1	1	1	1	1	1	1	1

Total cost: 3491 units.

c. Method 2 (TABU) solution.

					Skipping Pattern									
No.	Volume	Arrival	Depart	Stop:	1	2	3	4	5	6	7	8	9	10
1.	180	6.50	7.00		0	0	0	0	1	0	1	1	1	1
2.	180	7.40	8.00		1	1	1	0	0	1	1	1	1	1
3.	180	8.55	9.00		1	1	1	1	1	1	1	1	1	1

Total cost: 3617 units.

d. Method 3 solution.

6. CONCLUSION

We have presented a 0-1 stochastic programming model and methods of solution for the real-time bus scheduling problem for this problem. The methods were applied to data coming from the actual operation of trolley route 9 in Shanghai. The results obtained demonstrates the potential of the method for the improvement of transit operation in practice. Plans for future research include the extension of the model to allow for random travel times and an extensive analysis of real-time control strategies using a full scale simulation of a bus line operation.

REFERENCES

[1] Bly, P.H. and R.L. Jackson, "Evaluation of bus control strategies by simulation," Transportation and Road Research Laboratory, Crowthorne, U.K. Laboratory Report **637** (1974).

[2] Bodin, et al., "Survey of routing and scheduling of vehicles and crews," Computers & Operations Research, Vol. **10**, No. **2** (1983).

[3] Ceder, A., "Optimal design of transit short-turn," Transportation Research Record **1221** (1989) 8-22.

[4] Cury, J.E., A. C. Gomide and M. J. Mendes, "A methodology for generation of optimal schedules for an underground railway system," IEEE Transactions on Automatic Control, Vol. **Ac-25**, No. **2**, (1980) 217-222.

[5] Daduna, J. and A. Wren (eds.), Computer Aided Transit Scheduling, Springer-Verlag, Berlin (1988).

[6] Deckro, R.E., and E. P. Winkofsky, "Solving zero-one multiple objective programs through implicit enumeration," Eur. J. Oper. Res. (Netherlands) **12** (1983) 362-374.

[7] Desrochers, M.(ed.), Computer Scheduling of Public Transport **4**, to appeal.

[8] Glover, F., "Tabu search, part I", ORSA J. on Computing, Vol. 1, No. **3** (1989) 190-206.

[9] Glover, F., "Tabu search, part II", ORSA J. on Computing, Vol. 2, No. **1** (1990) 4-32.

[10] Koffman, D., "A simulation study of alternative real time bus headway control strategies," Transportation Research Record **663** (1978) 70-73.

[11] Li, Y. "The application of the microcomputer in bus and crew scheduling," in: Computer Scheduling of Public Transport **2**, Rousseau, J.-M. (ed.), Elsevier Science Publishers B.V. North-Holland (1985) 179-192.

[12] Li, Y., "Heuristic algorithm for large-scale zero-one goal programming problems," Theory and Practice of Systems Eng. **3** (1985) 73-75.

[13] Li, Y. , D. Kira and M. Kusy, "A heuristic approach to large-scale stochastic 0-1 linear programming with multiple objectives (SZOGP)", Presented in (October 1989) ORSA/TIMS Joint National Meeting, New York.

[14] Li, Y., M. Gendreau, "Approaches to stochastic 0-1 linear programming with simple recourse," Publication **#769**, Centre de Recherche sur les Transports, Univ. De Montreal (1991)

[15] Özekicii, S., "Average waiting time in queues with scheduled batch services," Transportation Science, Vol. **21**, No. **1** (1987) 55-61.

[16] Rousseau, J.-M.(ed.), Computer Scheduling of Public Transport **2**, Elsevier Science Publishers B.V. North-Holland (1985).

[17] Rousseau, J.-M. "Operations research in transit: a survey," Operational Research, G.K. Rand (ed.), Elsevier Science Publishers B.V. North-Holland (1988) 663-685.

[18] Ryan, K. T.,"Validating a bus operations simulation model," Winter Simulation Conference, San Deego, California (1979) 483-495.

[19] Scott, D., "Minimal fleet size in transhipment-type vehicle scheduling problems," Publication **#487**, Centre de Recherche sur les Transports, Univ. De Montreal (1986)

[20] Wren, A. (ed.), Computer Scheduling of Public Transport - Urban Passenger Vehicle and Crew Scheduling, Elsevier Science Publishers B. V. North-Holland (1981).